Chapter 10
(dun dun dunnnnnnnn)

Learing objectives: area and perimeter of basic polygons

Essential questions: how do you find the area and perimeter of basic objects?

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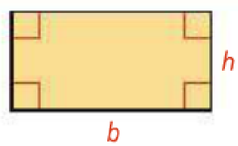
Back to basics!

take note

Theorem 10-1 Area of a Rectangle

The area of a rectangle is the product of its base and height.

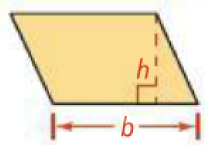
$A = bh$



Theorem 10-2 Area of a Parallelogram

The area of a parallelogram is the product of a base and the corresponding height.

$A = bh$



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Check this out!

Vocab!

A **base of a parallelogram** can be any one of its sides. The corresponding **altitude** is a segment perpendicular to the line containing that base, drawn from the side opposite the base. The **height** is the length of an altitude.

Where have we seen altitudes before?

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Example!

What is DE to the nearest tenth?

First, find the area of $\square ABCD$. Then use the area formula a second time to find DE.

$A = bh$

$= 13(9) = 117$ Use base AD and height CF.

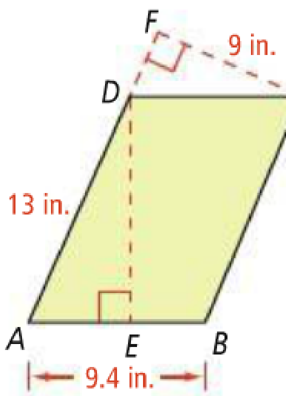
The area of $\square ABCD$ is 117 in.^{2}.

$A = bh$

$117 = 9.4(DE)$ Use base AB and height DE.

$DE = \frac{117}{9.4} \approx 12.4$

DE is about 12.4 in.



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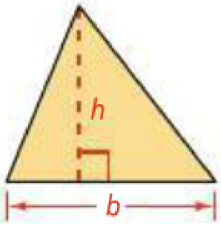
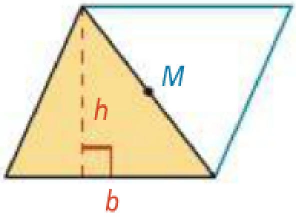
How this works...

You can rotate a triangle about the midpoint of a side to form a parallelogram.





The area of the triangle is half the area of the parallelogram.

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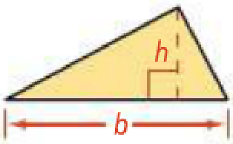
Theoretically speaking

Theorem 10-3 Area of a Triangle

The area of a triangle is half the product of a base and the corresponding height.

$$A = \frac{1}{2}bh$$



What is "h" referred to as?

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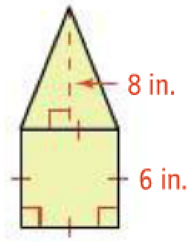
Divide and conquer!

How can you determine the area of the object to the right?



Find the area of each part of the figure.

triangle area $= \frac{1}{2}bh = \frac{1}{2}(6)8 = 24$ in.^{2}

square area $= bh = 6(6) = 36$ in.^{2}

area of the figure $= 24$ in.$^2 + 36$ in.$^2 = 60$ in.^{2}

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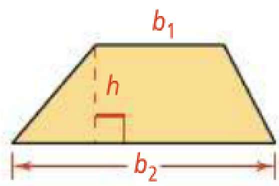
More theorems about shapes

Theorem 10-4 Area of a Trapezoid

The area of a trapezoid is half the product of the height and the sum of the bases.

$$A = \frac{1}{2}h(b_1 + b_2)$$



A trapezoid has four sides; only one pair is parallel!

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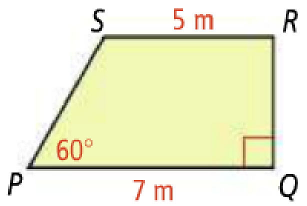
Check your altitude

How would you find the area of PQRS?



You can draw an altitude that divides the trapezoid into a rectangle and a 30°-60°-90° triangle. Since the opposite sides of a rectangle are congruent, the longer base of the trapezoid is divided into segments of lengths 2 m and 5 m.

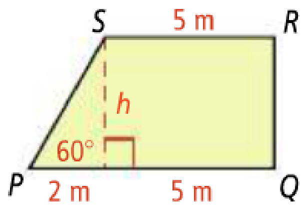
$h = 2\sqrt{3}$ longer leg = shorter leg • $\sqrt{3}$

$A = \frac{1}{2}h(b_1 + b_2)$ Use the trapezoid area formula.

$= \frac{1}{2}(2\sqrt{3})(7 + 5)$ Substitute $2\sqrt{3}$ for h, 7 for b_1, and 5 for b_2.

$= 12\sqrt{3}$ Simplify.

The area of trapezoid $PQRS$ is $12\sqrt{3}$ m^2.



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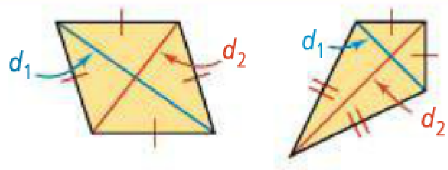
Designed Diagonally

Theorem 10-5 Area of a Rhombus or a Kite

The area of a rhombus or a kite is half the product of the lengths of its diagonals.

$$A = \frac{1}{2}d_1 d_2$$



Rhombus Kite

Note the difference in the equation!

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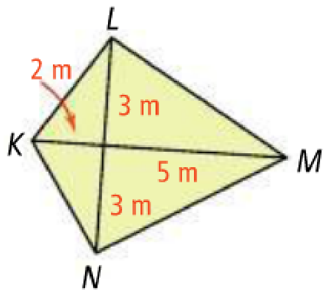
Q: How much area does a kite have?

Anything cross your mind about the area of this kite?



Find the lengths of the two diagonals:
$KM = 2 + 5 = 7$ m and $LN = 3 + 3 = 6$ m.

$A = \frac{1}{2}d_1 d_2$ Use the formula for area of a kite.

$= \frac{1}{2}(7)(6)$ Substitute 7 for d_1 and 6 for d_2.

$= 21$ Simplify.

The area of kite $KLMN$ is 21 m^2.

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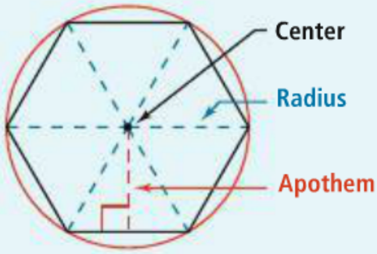
Generally speaking

Essential Understanding The area of a regular polygon is related to the distance from the center to a side.

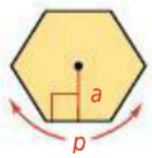
You can circumscribe a circle about any regular polygon. The center of a regular polygon is the center of the circumscribed circle. The **radius of a regular polygon** is the distance from the center to a vertex. The **apothem** is the perpendicular distance from the center to a side.



Theorem 10-6 Area of a Regular Polygon

The area of a regular polygon is half the product of the apothem and the perimeter.

$A = \frac{1}{2}ap$



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Obviously....

Postulate 10-1

If two figures are congruent, then their areas are equal.

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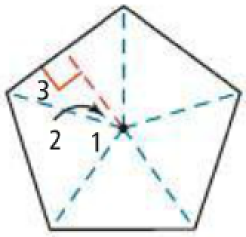
Angles in polygons

The figure at the right is a regular pentagon with radii and an apothem drawn. What is the measure of each numbered angle?



$m\angle 1 = \frac{360}{5} = 72$ Divide 360 by the number of sides.

$m\angle 2 = \frac{1}{2}m\angle 1$ The apothem bisects the vertex angle of the isosceles triangle formed by the radii.

$= \frac{1}{2}(72) = 36$

$90 + 36 + m\angle 3 = 180$ The sum of the measures of the angles of a triangle is 180.

$m\angle 3 = 54$

$m\angle 1 = 72$, $m\angle 2 = 36$, and $m\angle 3 = 54$.

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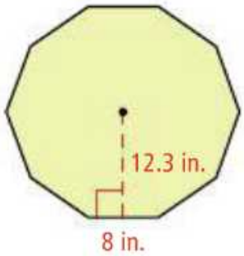
Example

What is the area of the regular decagon at the right?



Step 1 Find the perimeter of the regular decagon.

$p = ns$ Use the formula for the perimeter of an n-gon.

$= 10(8)$ Substitute 10 for n and 8 for s.

$= 80$ in.

Step 2 Find the area of the regular decagon.

$A = \frac{1}{2}ap$ Use the formula for the area of a regular polygon.

$= \frac{1}{2}(12.3)(80)$ Substitute 12.3 for a and 80 for p.

$= 492$

The regular decagon has an area of 492 in.^{2}.

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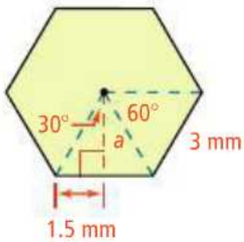
Example



Step 1 Find the apothem.

The radii form six 60° angles at the center, so you can use a 30°-60°-90° triangle to find the apothem.

$a = 1.5\sqrt{3}$ longer leg $= \sqrt{3} \cdot$ shorter leg

Step 2 Find the perimeter.

$p = ns$ Use the formula for the perimeter of an n-gon.

$= 6(3)$ Substitute 6 for n and 3 for s.

$= 18$ mm

Step 3 Find the area.

$A = \frac{1}{2}ap$ Use the formula for the area of a regular polygon.

$= \frac{1}{2}(1.5\sqrt{3})(18)$ Substitute $1.5\sqrt{3}$ for a and 18 for p.

≈ 23.3826859 Use a calculator.

The area is about 23 mm^2.

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